Extra Example 6

The Navy Pier Ferris wheel in Chicago has a diameter of 140 feet and its base is 10 feet off the ground. You board a gondola on the Ferris wheel and rotate 158° counterclockwise before the wheel temporarily stops. How high above the ground are you when the wheel stops? **about 145 ft**

Graphing Calculator

Make sure that students understand when they use their calculators in Example 6, they can enter sin 110° directly. It is not necessary to use reference angles when you are using a calculator.

Closing the Lesson

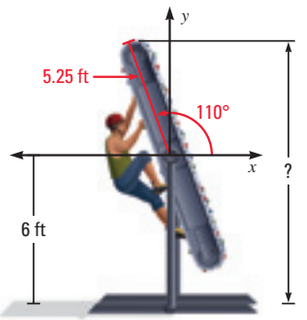
Have students summarize the major points of the lesson and answer the Essential Question: How can you evaluate trigonometric functions of any angle?

- **The six trigonometric functions can be defined for any angle, positive or negative, and measured in degrees or radians.**
- **The reference angle for an angle θ is the acute angle θ' formed by the terminal side of θ and the x-axis.**

If you know the coordinates of a point on the terminal side of the angle, use the general definitions of the six functions. For a quadrantal angle, use the unit circle. If the reference angle is 30°, 45°, or 60°, find the function value for the reference angle and adjust the sign as needed for the quadrant of the angle. If you cannot use any of these methods to find exact values, use a calculator to find approximate function values.

EXAMPLE 6 Model with a trigonometric function

ROCK CLIMBING A rock climber is using a rock climbing treadmill that is 10.5 feet long. The climber begins by lying horizontally on the treadmill, which is then rotated about its midpoint by 110° so that the rock climber is climbing towards the top. If the midpoint of the treadmill is 6 feet above the ground, how high above the ground is the top of the treadmill?



Solution

$\sin \theta = \frac{y}{r}$ — Use definition of sine.

$\sin 110° = \frac{y}{5.25}$ — Substitute 110° for θ and $\frac{10.5}{2} = 5.25$ for r.

$4.9 \approx y$ — Solve for y.

▶ The top of the treadmill is about $6 + 4.9 = 10.9$ feet above the ground.

GUIDED PRACTICE for Examples 5 and 6

10. TRACK AND FIELD Estimate the horizontal distance traveled by a track and field long jumper who jumps at an angle of 20° and with an initial speed of 27 feet per second. **about 14.64 ft**

11. WHAT IF? In Example 6, how high is the top of the rock climbing treadmill if it is rotated 100° about its midpoint? **about 11.2 ft**

13.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 17, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 33, 37, and 39

SKILL PRACTICE

A

1. VOCABULARY Copy and complete: A(n) ? is an angle in standard position whose terminal side lies on an axis. **reference angle**

2. ★ WRITING Given an angle θ in Quadrant III, explain how you can use a reference angle to find $\cos \theta$. ***Sample answer:*** **Subtract 180° from the angle measure to determine the reference angle. Then, compute the cosine of this angle and make it negative.**

EXAMPLE 1 on p. 866 for Exs. 3–11

USING A POINT **Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ.** **3–10. See margin.**

3. (8, 15) **4.** (−9, 12) **5.** (−7, −24) **6.** (5, −12)

7. (2, −2) **8.** (−6, 9) **9.** (−3, −5) **10.** $(5, -\sqrt{11})$

11. ★ MULTIPLE CHOICE Let (−7, −4) be a point on the terminal side of an angle θ in standard position. What is the value of $\tan \theta$? **C**

Ⓐ $-\frac{7}{4}$ Ⓑ $-\frac{4}{7}$ Ⓒ $\frac{4}{7}$ Ⓓ $\frac{7}{4}$

2–10. See Additional Answers beginning on p. AA1.

EXAMPLE 2 on p. 867 for Exs. 12–15

QUADRANTAL ANGLES **Evaluate the six trigonometric functions of θ.** 12–15. See margin.

12. $\theta = 0°$ **13.** $\theta = \frac{\pi}{2}$ **14.** $\theta = 540°$ **15.** $\theta = \frac{7\pi}{2}$

EXAMPLE 3 on p. 868 for Exs. 16–23

FINDING REFERENCE ANGLES **Sketch the angle. Then find its reference angle.** 16–23. See margin for art.

16. $-100°$ 80° **17.** 150° 30° **18.** 320° 40° **19.** $-370°$ 10°

20. $-\frac{5\pi}{6}$ $\frac{\pi}{6}$ **21.** $\frac{8\pi}{3}$ $\frac{\pi}{3}$ **22.** $\frac{15\pi}{4}$ $\frac{\pi}{4}$ **23.** $-\frac{13\pi}{6}$ $\frac{\pi}{6}$

EXAMPLE 4 B on p. 869 for Exs. 24–31

EVALUATING FUNCTIONS **Evaluate the function without using a calculator.**

24. sec 135° $-\sqrt{2}$ **25.** tan 240° $\sqrt{3}$ **26.** sin $(-150°)$ $-\frac{1}{2}$ **27.** csc $(-420°)$ $-\frac{2\sqrt{3}}{3}$

28. $\cos\frac{7\pi}{4}$ $\frac{\sqrt{2}}{2}$ **29.** $\cot\left(-\frac{8\pi}{3}\right)$ $\frac{\sqrt{3}}{3}$ **30.** $\tan\left(-\frac{3\pi}{4}\right)$ 1 **31.** $\sec\frac{11\pi}{6}$ $\frac{2\sqrt{3}}{3}$

32. **ERROR ANALYSIS** Let (4, 3) be a point on the terminal side of an angle θ in standard position. *Describe* and correct the error in finding $\tan\theta$.

$\tan\theta = \frac{x}{y} = \frac{4}{3}$ ✗

The equation for tangent is $\tan\theta = \frac{y}{x}$; $\tan\theta = \frac{3}{4}$.

C **33.** ★ **SHORT RESPONSE** Write $\tan\theta$ as the ratio of two other trigonometric functions. Use this ratio to explain why $\tan 90° = 0$. See margin.

34. **CHALLENGE** Five of the most famous numbers in mathematics — 0, 1, π, e, and i — are related by the simple equation $e^{\pi i} + 1 = 0$. Derive this equation using Euler's formula: $e^{a+bi} = e^a(\cos b + i\sin b)$. See margin.

PROBLEM SOLVING

EXAMPLE 5 A on p. 869 for Exs. 35–36

In Exercises 35 and 36, use the formula in Example 5 on page 869.

35. **FOOTBALL** You and a friend each kick a football with an initial speed of 49 feet per second. Your kick is projected at an angle of 45° and your friend's kick is projected at an angle of 60°. About how much farther will your football travel than your friend's football? about 10 ft

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36. **IN-LINE SKATING** At what speed must the in-line skater launch himself off the ramp in order to land on the other side of the ramp? about 16.5 ft/sec

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37. About 104 ft; no. *Sample answer:* The initial height that the Ferris wheel is above the ground is not doubled so the entire height is not doubled.

EXAMPLE 6 on p. 870 for Exs. 37–38

37. ★ **SHORT RESPONSE** A Ferris wheel has a radius of 75 feet. You board a car at the bottom of the Ferris wheel, which is 10 feet above the ground, and rotate 255° counterclockwise before the ride temporarily stops. How high above the ground are you when the ride stops? If the radius of the Ferris wheel is doubled, is your height above the ground doubled? *Explain*.

12. $\sin\theta = 0$, $\cos\theta = 1$, $\tan\theta = 0$, $\csc\theta$ = undefined, $\sec\theta = 1$, $\cot\theta$ = undefined
13. $\sin\theta = 1$, $\cos\theta = 0$, $\tan\theta$ = undefined, $\csc\theta = 1$, $\sec\theta$ = undefined, $\cot\theta = 0$
14. $\sin\theta = 0$, $\cos\theta = -1$, $\tan\theta = 0$, $\csc\theta$ = undefined, $\sec\theta = -1$, $\cot\theta$ = undefined
15. $\sin\theta = -1$, $\cos\theta = 0$, $\tan\theta$ = undefined, $\csc\theta = -1$, $\sec\theta$ = undefined, $\cot\theta = 0$
16–23. See Additional Answers beginning on p. AA1.

4 PRACTICE AND APPLY

Assignment Guide

Answer Transparencies available for all exercises

Basic:
Day 1: pp. 870–872
Exs. 1–15, 47–56
Day 2: pp. 870–872
Exs. 16–27, 35–38, 41–46

Average:
Day 1: pp. 870–872
Exs. 1, 2, 4–15, 32, 47–56
Day 2: pp. 870–872
Exs. 17–21, 25–29, 35–39, 41–46

Advanced:
Day 1: pp. 870–872
Exs. 1, 2, 5–15, 32, 33, 47–56
Day 2: pp. 870–872
Exs. 20–23, 28–31, 34–46*

Block:
pp. 870–872
Exs. 1, 2, 4–15, 17–21, 25–29, 32, 35–39, 41–56

Differentiated Instruction

See *Algebra 2 Best Practices Toolkit* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 12, 18, 24, 35
Average: 6, 14, 20, 28, 36
Advanced: 8, 15, 22, 30, 38

Extra Practice

- Student Edition, p. 1022
- Chapter 13 Resource Book: Practice levels A, B, C, pp. 27–29

Practice Worksheet

An easily-readable reduced practice page (with answers) for this lesson can be found on p. 850C.

33. $\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\sin 90° = 1$ and the $\cos 90° = 0$ so the $\tan 90°$ is undefined because you cannot divide by zero but the $\cot\theta = \frac{0}{1} = 0$.

34. See Additional Answers beginning on p. AA1.

5 ASSESS AND RETEACH

Daily Homework Quiz

Transparency Available

1. Let (8, −10) be a point on the terminal side of an angle θ in standard position. Evaluate $\cos\theta$. $\frac{4\sqrt{41}}{41}$
2. Use the unit circle to evaluate $\csc 540°$. **undefined**
3. Find the reference angle θ' for $\theta = \frac{11\pi}{8}$. $\theta' = \frac{3\pi}{8}$
4. Evaluate $\cot(-225°)$. **−1**
5. You kick a football at an initial speed of 50 feet per second, projected at an angle of 36°. About how far will the ball travel horizontally before hitting the ground? **about 74.3 ft**

Online Quiz

Available at **classzone.com**

Diagnosis/Remediation

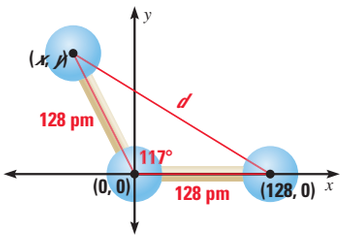
- Practice A, B, C in Chapter 13 Resource Book, pp. 27–29
- Study Guide in Chapter 13 Resource Book, pp. 30–31
- Practice Workbook, pp. 185–186
- @HomeTutor

Challenge

Additional challenge is available in the Chapter 13 Resource Book, p. 35.

B

38. MULTI-STEP PROBLEM When two atoms in a molecule are bonded to a common atom, chemists are interested in both the bond angle and the lengths of the bonds. An ozone molecule (O_3) is made up of two oxygen atoms bonded to a third oxygen atom, as shown.



a. In the diagram, coordinates are given in picometers (pm). (*Note:* 1 pm = 10^{-12} m.) Find the coordinates (x, y) of the center of the oxygen atom in Quadrant II. **(58.1, 114)**

b. Find the distance d (in picometers) between the centers of the two unbonded oxygen atoms. **about 134 pm**

39. ★ EXTENDED RESPONSE A sprinkler at ground level is used to water a garden. The water leaving the sprinkler has an initial speed of 25 feet per second.

a. Calculate Copy the table below. Use the formula in Example 5 on page 869 to complete the table.

Angle of sprinkler, θ	25°	30°	35°	40°	45°	50°	55°	60°	65°
Horizontal distance water travels, d	? 15.0	? 16.9	? 18.4	? 19.2	? 19.5	? 19.2	? 18.4	? 16.9	? 15.0

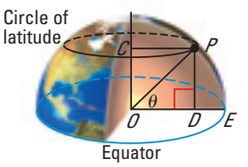
b. Interpret What value of θ appears to maximize the horizontal distance traveled by the water? Use the formula for horizontal distance traveled and the unit circle to explain why your answer makes sense.

c. Compare *Compare* the horizontal distance traveled by the water when $\theta = (45 - k)°$ with the distance when $\theta = (45 + k)°$. **The distances are the same.**

39b. 45°; since $\frac{v^2}{32}$ is constant in this situation, the maximum distance traveled will occur when sin 2θ is as large as possible. The maximum value of sine occurs when $\theta = 90°$, so $2\theta = 90°$, $\theta = 45°$.

C

40. CHALLENGE The latitude of a point on Earth is the degree measure of the shortest arc from that point to the equator. For example, the latitude of point P in the diagram equals the degree measure of arc PE. At what latitude θ is the circumference of the circle of latitude at P half the distance around the equator? **60°**



MIXED REVIEW

PREVIEW
Prepare for Lesson 13.4 in Exs. 41–46.

Graph the function f. Then use the graph to determine whether the inverse of f is a function. *(p. 438)* **41–46. See margin for art.**

41. $f(x) = 5x + 2$ **function**
42. $f(x) = -x + 7$ **function**
43. $f(x) = x^2 + 5$ **not a function**
44. $f(x) = 4x^2, x \geq 0$ **function**
45. $f(x) = 0.25x^2$ **not a function**
46. $f(x) = |x - 7|$ **not a function**

Find the range and standard deviation of the data set. *(p. 744)*

47. 3, 5, 2, 3, 7, 11, 8, 4 **9, about 2.87**
48. 18, 12, 15, 9, 13, 7, 4, 17 **14, about 4.59**
49. 5.9, 8.2, 3.7, 6.1, 2.9, 1.8, 5.7 **6.4, about 2.04**
50. 54, 60, 57, 53, 59, 51, 56, 62 **11, about 3.5**

Find the sum of the series.

51. $\sum_{i=1}^{15}(3i + 2)$ *(p. 802)* **390**
52. $\sum_{i=1}^{18}(4i + 1)$ *(p. 802)* **702**
53. $\sum_{i=1}^{24}(17 - 2i)$ *(p. 802)* **−192**
54. $\sum_{i=1}^{5} 2(3)^{i-1}$ *(p. 810)* **242**
55. $\sum_{i=1}^{7} \frac{1}{4}\left(\frac{3}{2}\right)^{i-1}$ *(p. 810)* $8\frac{11}{256}$
56. $\sum_{i=1}^{\infty} 8\left(\frac{1}{2}\right)^{i-1}$ *(p. 820)* **16**

EXTRA PRACTICE for Lesson 13.3, p. 1022 **ONLINE QUIZ** at classzone.com

41–46. See Additional Answers beginning on p. AA1.